

Objective: TSWBAT - use properties of math to  
simplify expressions

Take note

## Properties Properties of Real Numbers

Let  $a$ ,  $b$ , and  $c$  be any real numbers.

### Commutative Properties of Addition and Multiplication

Changing the order of the addends does not change the sum. Changing the order of the factors does not change the product.

	Algebra	Example
<b>Addition</b>	$a + b = b + a$	$18 + 54 = 54 + 18$
<b>Multiplication</b>	$a \cdot b = b \cdot a$	$12 \cdot \frac{1}{2} = \frac{1}{2} \cdot 12$

### Associative Properties of Addition and Multiplication

Changing the grouping of the addends does not change the sum. Changing the grouping of the factors does not change the product.

<b>Addition</b>	$(a + b) + c = a + (b + c)$	$(23 + 9) + 4 = 23 + (9 + 4)$
<b>Multiplication</b>	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(7 \cdot 9) \cdot 10 = 7 \cdot (9 \cdot 10)$



## Properties of Real Numbers

Let  $a$  be any real number.

### Identity Properties of Addition and Multiplication

The sum of any real number and 0 is the original number. The product of any real number and 1 is the original number.

	Algebra	Example
<b>Addition</b>	$a + 0 = a$	$5\frac{3}{4} + 0 = 5\frac{3}{4}$
<b>Multiplication</b>	$a \cdot 1 = a$	$67 \cdot 1 = 67$

### Zero Property of Multiplication

The product of  $a$  and 0 is 0.

$a \cdot 0 = 0$	$18 \cdot 0 = 0$
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### Multiplication Property of $-1$

The product of  $-1$  and  $a$  is  $-a$ .

$-1 \cdot a = -a$	$-1 \cdot 9 = -9$
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## Properties Properties of Real Numbers

Draw a line from each property in Column A to the equation that illustrates it in Column B.

### Column A

6. Associative Property of Addition
7. Associative Property of Multiplication
8. Commutative Property of Addition
9. Commutative Property of Multiplication
10. Identity Property of Addition
11. Identity Property of Multiplication
12. Multiplication Property of  $-1$
13. Zero Property of Multiplication

### Column B

- A)  $15y + 0 = 15y$
- B)  $7b \cdot 2 = 2 \cdot 7b$
- C)  $(c \cdot 3) \cdot 5 = c \cdot (3 \cdot 5)$
- D)  $6x + 5y = 5y + 6x$
- E)  $a \cdot 1 = a$
- F)  $(g + 11h) + 9h = g + (11h + 9h)$
- G)  $7k \cdot 0 = 0$
- H)  $15m \cdot (-1) = -15m$



### Problem 1 Identifying Properties

What property is illustrated by each statement?

**A**  $42 \cdot 0 = 0$  Zero Property of Multiplication

**B**  $(y + 2.5) + 28 = y + (2.5 + 28)$  Associative Property of Addition

**C**  $10x + 0 = 10x$  Identity Property of Addition



**Got It?** 1. What property is illustrated by each statement?

a.  $4x \cdot 1 = 4x$

b.  $x + (\sqrt{y} + z) = x + (z + \sqrt{y})$

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### Problem 1 Identifying Properties

**Got It?** What property is illustrated by  $4x \cdot 1 = 4x$ ?

14. For each question, determine if the stated characteristic is or is not being illustrated by  $4x \cdot 1 = 4x$ .

Is the same number being added to both sides of the equation?

Yes / No

Are groupings being changed in the equation?

Yes / No

Is 0 or 1 part of the equation?

Yes / No

15. Think of the operation symbol that will make the equation  $4x \blacksquare 1 = 4x$  true.  
What property is illustrated by  $4x \cdot 1 = 4x$ ?

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You can use properties to help you solve some problems using mental math.



### **Problem 2** Using Properties for Mental Calculations

**Movies** A movie ticket costs \$7.75. A drink costs \$2.40. Popcorn costs \$1.25. What is the total cost for a ticket, a drink, and popcorn? Use mental math.

$$\begin{aligned}(7.75 + 2.40) + 1.25 &= (2.40 + 7.75) + 1.25 && \text{Commutative Property of Addition} \\ &= 2.40 + (7.75 + 1.25) && \text{Associative Property of Addition} \\ &= 2.40 + 9 && \text{Simplify inside parentheses.} \\ &= 11.40 && \text{Add.}\end{aligned}$$

The total cost is \$11.40.



**Got It?** 2. A can holds 3 tennis balls. A box holds 4 cans. A case holds 6 boxes. How many tennis balls are in 10 cases? Use mental math.



### Problem 3 Writing Equivalent Expressions

Simplify each expression.

**A**  $5(3n)$

**Know**

An expression

**Need**

Groups of numbers that can be simplified

**Plan**

Use properties to group or reorder parts of the expression.

$$\begin{aligned} 5(3n) &= (5 \cdot 3)n && \text{Associative Property of Multiplication} \\ &= 15n && \text{Simplify.} \end{aligned}$$

**B**  $(4 + 7b) + 8$

$$\begin{aligned} (4 + 7b) + 8 &= (7b + 4) + 8 && \text{Commutative Property of Addition} \\ &= 7b + (4 + 8) && \text{Associative Property of Addition} \\ &= 7b + 12 && \text{Simplify.} \end{aligned}$$

**C**  $\frac{6xy}{y}$

$$\begin{aligned} \frac{6xy}{y} &= \frac{6x \cdot y}{1 \cdot y} && \text{Rewrite denominator using Identity Property of Multiplication.} \\ &= \frac{6x}{1} \cdot \frac{y}{y} && \text{Use rule for multiplying fractions: } \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}. \\ &= 6x \cdot 1 && x \div 1 = x \text{ and } y \div y = 1. \\ &= 6x && \text{Identity Property of Multiplication} \end{aligned}$$



**Got It?** 3. Simplify each expression.

a.  $2.1(4.5x)$

b.  $6 + (4h + 3)$

c.  $\frac{8m}{12mn}$



### Problem 3 Writing Equivalent Expressions

**Got It?** Simplify each expression.

$2.1(4.5x)$

$6 + (4h + 3)$

$\frac{8m}{12mn}$

In Exercises 19–20, each expression is simplified. Justify each step.

19.  $2.1(4.5x) = (2.1 \cdot 4.5)x$

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$= 9.45x$

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20.  $6 + (4h + 3) = (4h + 3) + 6$

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$= 4h + (3 + 6)$

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$= 4h + 9$

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21. Complete each step of the simplification.

$$\frac{8m}{12mn} = \frac{2 \cdot \square \cdot \square \cdot 1}{3 \cdot 4 \cdot m \cdot n}$$

$$= \frac{2}{3} \cdot \frac{\square}{\square} \cdot \frac{\square}{\square} \cdot \frac{1}{n} = \frac{2}{3} \cdot \square \cdot \square \cdot \frac{1}{n} = \frac{2}{3} \cdot \square = \square$$

## ● Vocabulary Builder

**counterexample** (noun) KOWN tur eg ZAM pul

**Definition:** A counterexample is an example that shows that a statement is not always true.

**Related Words:** counteract (verb), counterargument (noun), counterclockwise (adjective)

**Example:** For all real numbers,  $a + b = a \cdot b$  is a *false* statement. You can show the statement is false by using a **counterexample** like the one below.

$$5 + 3 = 8 \text{ is not equal to } 5 \cdot 3 = 15.$$

You need only **one counterexample** to prove that a statement is false.

## ● Use Your Vocabulary

Draw a line from each statement to a *counterexample* that shows it to be false.

Statement	Counterexample
3. If you live in Miami, you live in Florida.	Mexico is in North America.
4. If you live near an ocean, you live near the Atlantic Ocean.	People in California live near the Pacific Ocean.
5. If you live in North America, you live in the United States.	Miami is a city in Ohio.



### Problem 4 Using Deductive Reasoning and Counterexamples

**Got It? Reasoning** Is the statement *true* or *false*? If it is false, give a counterexample. If true, use properties of real numbers to show the expressions are equivalent.

For all real numbers  $j$  and  $k$ ,  $j \cdot k = (k + 0) \cdot j$ .

22. Simplify the right side of the equation above and state the property that you used.

$$(k + 0) \cdot j = \square \cdot j$$

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23. Complete: The simplified expression is equal to  $j \cdot k$  by the ? Property of Multiplication.

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24. So, the statement  $j \cdot k = (k + 0) \cdot j$  is ?.

True / False

**Got It? Reasoning** Is the statement *true* or *false*? If it is false, give a counterexample. If true, use properties of real numbers to show the expressions are equivalent.

For all real numbers  $m$  and  $n$ ,  $m(n + 1) = mn + 1$ .

Evaluate each expression for  $m = 4$  and  $n = 5$ .

25.  $m(n + 1) = \square \cdot (\square + 1)$   
 $= \square \cdot \square$   
 $= \square$

26.  $mn + 1$

27. Is the value of the expression in Exercise 25 equal to the value of the expression in Exercise 26?

Yes / No

28. Is the original statement *true* or *false*? If it is false, give a counterexample. If true, use properties of real numbers to show the expressions are equivalent.

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## Lesson Check • Do you UNDERSTAND?

Justify each step to show that  $3 \cdot (10 \cdot 12) = 360$ .

29. The left side of the expression is simplified below. Write a reason for each step.

$$3 \cdot (10 \cdot 12) = 3 \cdot (12 \cdot 10)$$

$$= (3 \cdot 12) \cdot 10$$

$$= 36 \cdot 10$$

$$= 360$$


## 1-4

## Practice

Form G

## Properties of Real Numbers

Name the property that each statement illustrates.

1.  $12 + 917 = 917 + 12$

2.  $74.5 \cdot 0 = 0$

3.  $35 \cdot x = x \cdot 35$

4.  $3 \cdot (-1 \cdot p) = 3 \cdot (-p)$

5.  $m + 0 = m$

6.  $53.7 \cdot 1 = 53.7$

Use mental math to simplify each expression.

7.  $36 + 12 + 4$

8.  $19.2 + 0.6 + 12.4 + 0.8$

9.  $2 \cdot 16 \cdot 10 \cdot 5$

10.  $12 \cdot 18 \cdot 0 \cdot 17$

Simplify each expression. Justify each step.

11.  $6 + (8x + 12)$

12.  $5(16p)$

13.  $(2 + 7m) + 5$

14.  $\frac{12st}{4t}$

Tell whether the expressions in each pair are equivalent.

15.  $7x$  and  $7x \cdot 1$

16.  $4 + 6 + x$  and  $4 \cdot x \cdot 6$

17.  $(12 - 7) + x$  and  $5x$

18.  $p(4 - 4)$  and  $0$

19.  $\frac{24xy}{2x}$  and  $12y$

20.  $\frac{27m}{(3+9) \cdot 12}$  and  $27m$

21. You have prepared 42 mL of distilled water, 18 mL of vinegar and 47 mL of salt water for an experiment.
- How many milliliters of solution will you have if you first pour the distilled water, then the salt water, and finally the vinegar into your beaker?
  - How many milliliters of solution will you have if you first pour the salt water, then the vinegar, and finally the distilled water into your beaker?
  - Explain why the amounts described in parts (a) and (b) are equal.

# 1-4 Practice (continued)

## Properties of Real Numbers

Form G

Use deductive reasoning to tell whether each statement is *true* or *false*. If it is false, give a counterexample.

22. For all real numbers  $a$  and  $b$ ,  $a - b = -b + a$ .

23. For all real numbers  $p$ ,  $q$  and  $r$ ,  $p - q - r = p - r - q$ .

24. For all real numbers  $x$ ,  $y$  and  $z$ ,  $(x + y) + z = z + (x + y)$ .

25. For all real numbers  $m$  and  $n$ ,  $\frac{m}{-n} = \frac{n}{-m}$ .

26. **Writing** Explain why the commutative and associative properties don't hold true for subtraction and division but the identity properties do.

27. **Reasoning** A recipe for brownies calls for mixing one cup of sugar with two cups of flour and 4 ounces of chocolate. They are all to be mixed in a bowl before baking. Will the brownies taste different if you add the ingredients in different orders? Relate your answer to a property of real numbers.

Simplify each expression. Justify each step.

28.  $(6^7)(5^3 + 2)(2 - 2)$

29.  $(m - 16)(-7 \div -7)$

30. Open-Ended Provide examples to show the following.

- a. The associative property of addition holds true for negative integers.
- b. The commutative property of multiplication holds true for non-integers.
- c. The multiplicative property of negative one holds true regardless of the sign of the number on which the operation is performed.
- d. The commutative property of multiplication holds true if one of the factors is zero.

## Exercises

What property is illustrated by each statement?

1.  $(m + 7.3) + 4.1 = m + (7.3 + 4.1)$

2.  $5p \cdot 1 = 5p$

3.  $12x + 4y + 0 = 12x + 4y$

4.  $(3r)(2s) = (2s)(3r)$

5.  $17 + (-2) = (-2) + 17$

6.  $-(-3) = 3$

Simplify each expression. Justify each step.

7.  $(12 + 8x) + 13$

8.  $(5 \cdot m) \cdot 7$

9.  $(7 - 7) + 12$

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# 1-4

## Enrichment

### Properties of Real Numbers

Which of the properties of real numbers are illustrated by the following situations? Explain your reasoning.

1. One team scores 3 runs in the first inning and 2 runs in the fourth inning. The other team scores 2 runs in the first inning and 3 runs in the fourth. In the fifth inning, the score is tied.
2. Your friend gets a job making \$9.50 per hour. One week she takes a vacation and does not work. She makes no money that week.
3. In putting together a mixture of fertilizer, a gardener mixes nitrogen and phosphorus before adding potassium. The next day the gardener mixes phosphorus and potassium before adding nitrogen. The two mixtures are exactly the same.
4. A restaurant received two orders from the apartment managers of two different apartment buildings. The first apartment manager said he was ordering 3 meals each for the occupants of 4 different apartments. The second said he was ordering 4 meals each for the occupants of 3 different apartments. The apartment managers ordered the same number of meals.
5. The owner of a theater checked how much money was in the box office 10 minutes before a show began. No tickets were purchased in the last 10 minutes, so the owner was not surprised that the final amount of money was the same as when when he previously checked.
6. Usually, when Marty makes pancakes for his kids, he changes the amount of each ingredient depending on how many servings he is making. Since he was making the exact number of servings the recipe called for, he was able to use the numbers published in the cook book.