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Comparing Kisses

MIDDLE GRADES STUDENTS SHOULD be given ample opportunities to explore probability and statistics topics. In particular, students should be encouraged to collect, organize, and describe data; construct appropriate charts and graphs to summarize the data; make predictions and conclusions based on the data; and test these predictions and conclusions (NCTM 1989, 2000). In this article, we describe three activities, all of which involve materials that students are certain to enjoy—Hershey's Kisses candies.

Activity 1—Which Is Your Favorite Kiss?

STUDENTS SHOULD HAVE HAD EXPERIENCE with collecting, organizing, and representing sets of data prior to the middle grades (NCTM 2000). Activity 1 allows students additional practice with these concepts. In particular, students review the importance of properly formulating questions and constructing and analyzing bar graphs.

To begin the lesson, we tell our class that we want to know which variety of Hershey's Kisses candies students like the most. We review the varieties of Hershey's candy—Kisses, Kisses with Almonds, and Hugs (which we will refer to in this article as plain, almond, and hugs, respectively), and tell students to work in groups to construct a survey that can be used to determine the favorite type. After the groups construct surveys, we ask them to share their ideas with the class. The survey question is consistently similar to "Which of the following Hershey's Kisses is your favorite?" and the three varieties of candies are listed. We discuss advantages and disadvantages of this type of question and refine it accordingly. A typical improvement to the list of choices is to include a "none of these" or "I don't know" for students who either do not eat chocolate or have not eaten this type of candy.

After we complete constructing the survey, we then survey the students in class and ask them to mark their response on a sticky note. They then place their notes appropriately on the whiteboard. Typical class data are shown in **figure 1.** Students are then asked to use the class data to construct a bar graph on grid paper. We remind students to use appropriate scales and to include headings and a title.

After students construct their bar graphs, we ask them to make conjectures based on the graphed data. For example, does the data indicate that almond Kisses are the favorite candy for most people in the school? In the state? Can we use the data to make any statements about adults' favorite variety of candy? Students are encouraged to justify their conjectures.

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Activity 2—What Is the Probability of a Kiss?

THE MIDDLE-GRADES MATHEMATICS CURRICULUM should include explorations of probability in real-world situations so that students can model them by devising and carrying out experiments or simulations to determine probabilities (NCTM 1989). In Activity 2, students explore the experimental probability of a plain Kiss landing on its flat base when tossed. The activity works best if students have experience calculating measures of central tendency, including mean, median, and mode. Students work in groups of three. Each group has ten plain Kisses, a 16-ounce plastic cup, and a flat table or desktop on which to work; each student has a sticky note and a copy of the **Activity 2 Worksheet.**

To begin the lesson, each student examines one plain Kiss. We discuss the possible outcomes if the candy was tossed onto the table. "It can land upright" (on its base) and "It can land sideways" are the two common responses. However, another enthusiastic response has included this statement: "I could toss it just right and it could just happen to land in my mouth!" For the purpose of this experiment, the class should agree that there are two possible outcomes for tossing the candy—landing on the base or landing on its side.

Each student is then asked to estimate the probability that a plain Kiss will land on its base when tossed. Students should write down and share their estimates and the reasoning behind them. Estimates usually range from 20 percent to 80 percent. "There's more room on the sides," explained a student who selected 40 percent as her probability, "it is more likely to land there than on the bottom." "It just seems like it will tip over more often and land on its side," said another. A student who selected a probability higher than 50 percent reasoned, "The base is all flat, so it will be more likely to land there and stay." It is important to note that, even though it is agreed that there are two outcomes, very few students select exactly 50 percent as their estimate. Students explain that there are two outcomes, but it does not seem like they will happen with the same frequency.

After students share their expected outcomes, they are ready to test them by conducting an experiment, as described on the **Activity 2 Worksheet.** We agree that, rather than toss candies up in the air as we would a coin, we will spill them from a cup. Each group puts all ten candies into a plastic cup, and the following tasks are assigned:

- One group member spills the candies onto the table ten times, each time counting the ones that land on their bases.
- A second group member helps the spiller count the candies.
- The third group member records the results on the **Activity 2 Worksheet.**

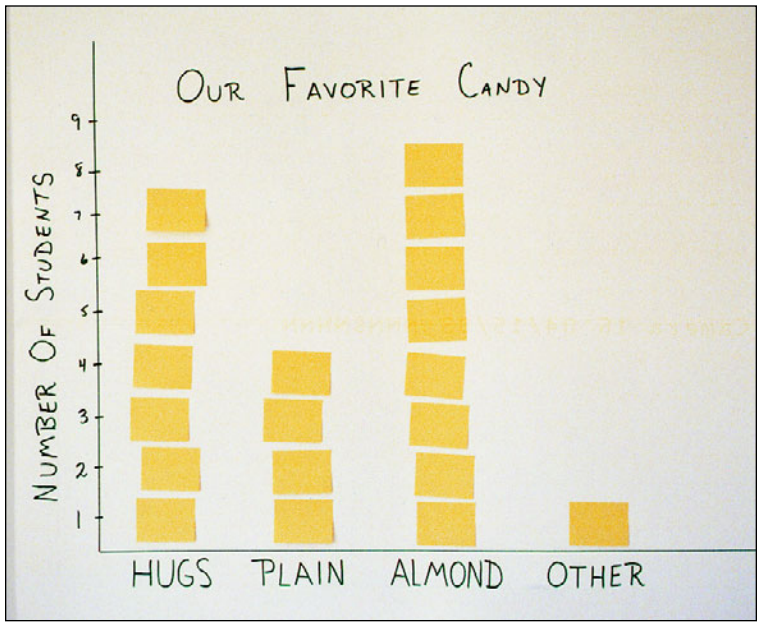

Fig. 1 Class data for Kisses survey

After one student has spilled the cup ten times, the assigned duties rotate among group members.

After all students have had a chance to conduct the experiment, we ask them to refine their original guesses. Many use only the total number of base landings from their own experiment, whereas others combine their total with those of the other members of their group. When asked how they could be more certain of the probabilities, most respond that a longer experiment (more tosses) would result in more certainty. However, students recognize that it is unrealistic for each of them to spill the candies for a very large number of times. Instead, we decide to use the results from the entire class to get a better estimate for the probability. We draw a number line, scaled according to the high and low results, on the whiteboard. Each student writes his or her own total number of base landings on a sticky note and posts the note appropriately to form a dot plot. After we discuss the data for the whole class, everyone makes a new estimate for their probability.

After we discuss class data, everyone makes a new estimate for their probability

Students should be asked to share their new probability estimates. Most students calculate the relative frequency (experimental probability) of

TABLE 1
Example Results for the Tosses of Kisses by One Student

Toss Number	Number of Plain Kisses Landing on Base	Number of Almond Kisses Landing on Base
1	7	4
2	3	5
3	2	4
4	4	3
5	8	5
6	4	4
7	4	3
8	5	3
9	3	3
10	3	2
Total	43	36

base landings for the class and use this value as their estimate. Others calculate the relative frequency and adjust the value to get an answer that they think will be more reasonable. For example, one student adjusted the experimental probability of 37 percent to 40 percent, explaining that she was more comfortable assigning an "even" number and thought it should be higher than 37 percent. Other students select the median or mode of the base landings for the class as their new estimate. We have conducted this experiment many times. In past experiments, the experimental probability for a tossed plain Kiss landing on its base has consistently been near 35 percent. Of course, differences in desktops, temperatures, and the very nature of the experiment may produce varying results.

Students should note the importance of testing these inferences when possible

The important concept in this activity is that one cannot simply count outcomes for every object and assign a theoretical probability to any given outcome. By exploring objects that cannot be assigned theoretical probabilities, students seem to have a greater understanding of when a theoretical probability can be assigned. Students recognize the need to generate data to estimate probabilities.

Activity 3—Comparing Kisses

IN GRADES 5–8, THE MATHEMATICS CURRICULUM should include explorations of probability in real-world situations so that students can make predictions about probabilities (NCTM 1989). Activity 3 allows students to make predictions about the probability of *almond* Kisses landing on their bases when spilled from a cup, after having experimented with plain Kisses. The activity works best if students have experience with five-number summaries (minimum, first quartile, median, third quartile, and maximum) and box plots (or box-and-whiskers plots); otherwise, instruction about these topics must be included in the lesson. Students continue working in groups of three. In addition to the materials used in the second activity, each group should have ten almond Kisses, and each student should have two sticky notes, each of a different color, and a copy of the **Activity 3 Worksheet.**

Each student compares one plain Kiss with one almond Kiss and notes any differences between them. Students usually notice that the wrapper colors differ and that the base of the plain candy is smaller, its edges are more tapered than the base of the almond candy, and it seems to be more slender than the almond candy. Students note that the wrapper color is not likely to affect the probability of the candy being able to land on its base, but the shape of the candy and the size of the base might change the data. Further, students note that the almond candy contains an almond, but it is difficult to determine how this might affect the outcome.

Students are asked to estimate the probability that an almond Kiss will land on its base when spilled in the same manner as the plain Kiss in activity 2. Almost without exception, students estimate that the almond candy is more likely than the plain candy to land on its base. Estimates for the almond candy usually range from 40 percent to 80 percent. Students cite the base size and candy shape as reasons for their conjecture.

Students are now ready to test their hypothesis that the almond Kiss will land on its base more often than the plain Kiss. *All twenty candies* should be placed in the cup, gently shaken, and spilled onto the desktop. We stress that all twenty candies should be spilled at the same time so that each type of candy is tossed in the same manner. One student should spill the candy, a second student should count the number of plain and almond candies that land on their bases, and the third student should record the results for each type of candy. After one student has spilled the candies ten times, students should rotate responsibilities. When each group member has completed the experiment, each student should total the number of times (out of 100) that each type of candy landed on its base. In **table 1,** we have included an example of typical outcomes for the tosses of the plain and almond Kisses.

Students should write their totals on sticky notes (agree ahead of time which color represents each

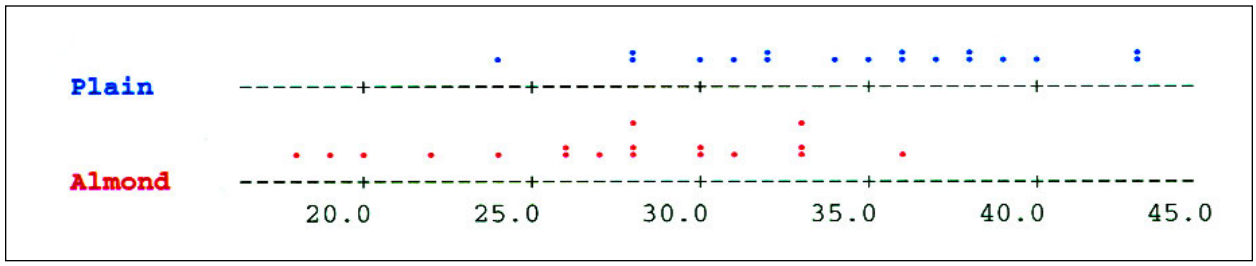

Fig. 2 Dot plots for example class data

candy) and post them on the whiteboard to create two dot plots on the same scale (see **fig. 2**). We have conducted this experiment many times. Each time, the plain candies land on their base more often than the almond candies. In past experiments, the probability for a tossed almond Kiss landing on its base has consistently been near 30 percent. Students get a feel for this counterintuitive result by looking at the class data, and the focus now shifts to what questions can be answered by analyzing the class results. Students work in their groups to find the five-number summaries of the class data for each type of candy. (See **table 2** for class example data.)

Next, students are asked to construct box plots of the class data. Within each type of candy, the box plots can be used to make a class decision as to what should be the claimed percentage of base landings. Between the two types of candy, the box plots can be used to compare the percentages of base landings. Comparative box plots for the example class data are displayed in **figure 3.** As shown on the **Activity 3 Worksheet,** we ask students to answer a series of questions to help summarize their findings.

The most surprising aspect of this activity is the result of a lower percentage of base landings for the almond candies. This situation illustrates how results that apply to one subgroup should not be generalized or adjusted to make inferences about a similar, but different subgroup. At a minimum, students should note the importance of testing these inferences, whenever possible.

TABLE 2
Five-Number Summaries for Class Example

Summary	Plain	Almond
Minimum	24%	18%
Quartile 1	31%	24%
Median	35.5%	28%
Quartile 3	38%	31%
Maximum	43%	36%

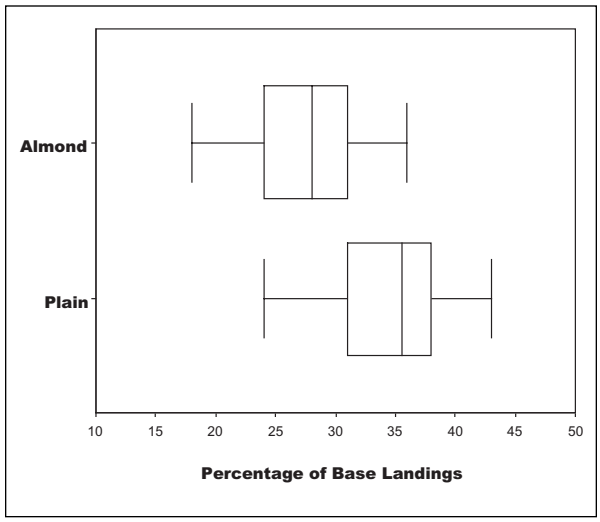

Fig. 3 Box plots for example class data

Discussion

THESE ACTIVITIES ARE WELL RECEIVED BY STUdents and are a great way to bring life to a discussion of several basic statistics topics. The candies are a motivating factor—students are very interested in analyzing the data in these activities, much more so than with other types of data.

Students not only enjoy the activities, but they often ask interesting and relevant follow-up questions that teachers may want to pursue. For example, students often want to know whether they would get similar results if they spill the candies from a higher point or spill the candies across the desk or table rather than in one spot. They also wonder whether the number of candies in the cup might influence the outcome of the experiment. These questions can be answered only by constructing and conducting appropriate experiments. We encourage teachers and students to do so!

References

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM, 1989.

———. *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000. □

(Worksheets follow on the next page.)

Activity 2 Worksheet

NAME _____

Question: What percent of the plain Hershey's Kiss chocolate candies land on their flat bases when spilled from a cup?

1. Initial guess: _____

During this activity, we will conduct an investigation to help us determine the chances of a plain Hershey's Kiss landing on its base when spilled from a cup. Your cup contains 10 plain candy Kisses. The investigation is as follows:

- Gently shake the cup a few times to help mix up the 10 candies.
- Tip the cup so that the bottom of the rim is approximately 1–2 inches from the table and spill all of the candies onto the table.
- Count the candies that land on their bases. Record this information in the table below.
  Return the candies to the cup and repeat the process until you have spilled the candies 10 times.

2. Use the table below to record your data. Find the total number of candies that landed on their bases.

Toss Number	Number of Kisses Landing on Base
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

3. Have others in the group perform the same experiment.

4. Make a new guess as to the percent of plain Hershey's Kisses that will land on their flat bases when spilled from a cup.

Updated guess: _____

5. How can you become more confident in your guess?

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Activity 3 Worksheet

NAME _____

Question: Will different types of Hershey's Kisses land on their bases at about the same rate?

1. Compare 1 plain and 1 almond Kiss. Note the differences in the candies. Based on your examination and what you already know about the plain candies, make an estimate as to the percent of almond candies that will land on their bases. Estimate: _____ How confident are you in this estimate?

The purpose of this activity is to help us compare the percent of times the candies will land on their bases when spilled from a cup. The investigation is as follows:

- Place **all 20** candies in the cup (10 of each).
- Gently shake the cup a few times to help mix up the candies. Spill all 20 of the candies onto the table.
- Count the plain and almond candies that land on their bases. Record this information in the table below.
- Return the candies to the cup, and repeat until you have spilled the candies 10 times.

2. Have others in the group perform the same experiment.

3. Use the table (below) and the box plot (on the next page) to record your data. Find the total number of each candy that landed on its base.

Toss Number	Number of **Plain** Kisses (silver foil) Landing on Base	Number of **Almond** Kisses (gold foil) Landing on Base
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
Total		

Class Data

Plain _____ Almond _____

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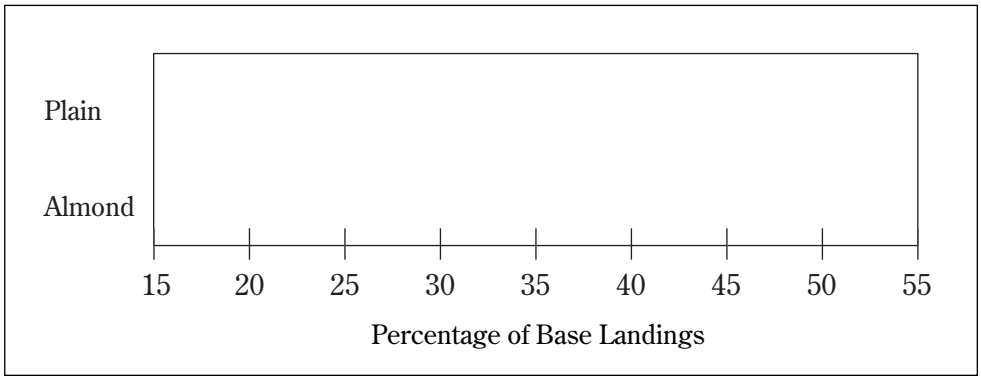
Activity 3 Worksheet *(continued)*

NAME _____

Percentiles of Five-Number Summaries

	PLAIN	ALMOND
Minimum		
Quartile 1		
Median		
Quartile 3		
Maximum		

Box Plot



3. Why is it important that the 10 plain and 10 almond candies be spilled at the same time?

4. How do the five-number summaries for the two data sets compare?

5. From the box plots, which type of Kiss appears to land on its base more often?

6. From the box plots, which type of Kiss has less variability in the number of base landings?

7. When you estimated the probability for the almond candies, you made a generalization based on what you knew about the plain candies. Why is it important to test generalizations whenever possible?

8. Do you think the results of this experiment would change if we were to increase or decrease the number of tosses? Explain your reasoning.

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