# Mathematics Detective

KATHY DAYMUDE

# What's My Number?

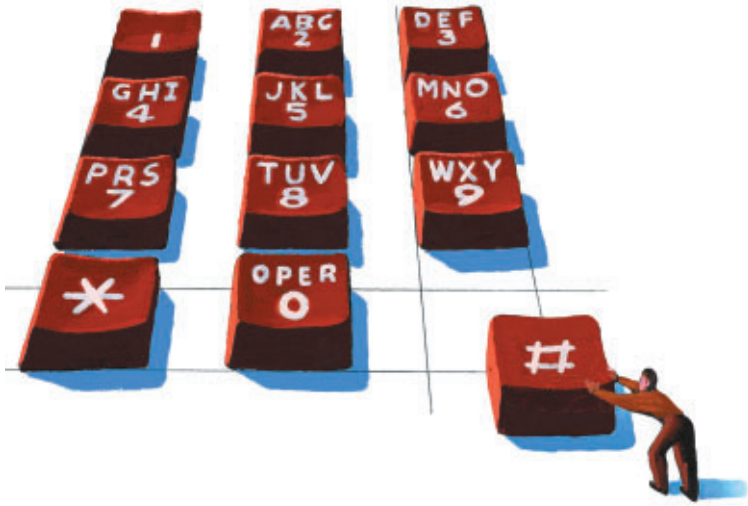
THE WAY WE MAKE TELEPHONE CALLS HAS changed many times since telephones were invented. The first telephones did not have numbers because the telephone companies thought that numbers would be too impersonal. Operators memorized the names of the customers and the position of their lines on a switchboard. During a measles epidemic in Lowell, Massachusetts, in 1879, the town doctor, Moses Greely Parker, recommended that numbers be assigned to the 200 phone customers so that if the town's four operators got sick, someone could take their places and prevent phone service from coming to a halt.

The first telephone numbers were often only four digits long. However, as more people got telephones, more numbers were needed. Telephone numbers evolved to consist of two letters linked to a small geographic area and five digits after that. My grandmother's number was SK3-0880, or Skyline 3-0880, because she lived in a neighborhood off Skyline Drive. If you check your phone, you will see that no letters are assigned to the numbers 1 and 0. What special meaning do these two digits have at the beginning of a phone number?

Around 1960, abandoning the two-letter beginnings in favor of two numbers extended the seven-digit phone numbers. This change freed up number pairs representing unintelligible letter combinations, such as the pair 97, and allowed phone numbers to have a 0 or a 1 as the second digit.

In the early days of telephone use, people could not dial long-distance numbers themselves but had to have the operator put the call through. In the mid-1940s, area codes were introduced in the United States as internal codes for the operators to use to route long-distance calls. In 1951, residents of Englewood, New Jersey, were the first to be able to dial long distance directly using area codes without operator assistance.

Let's investigate some mathematics of telephone numbers.




## Questions

1. How many phone numbers were possible using the early method of assigning a phone number beginning with 2 letters followed by 5 numbers? Explain how you found your answer.

2. How many 7-digit phone numbers were possible after 1960 when only numbers were used? How many phone numbers were added by this change? Does this system give enough phone numbers for your town or city? For your state? Explain.

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**3.** In 1947, the North American Numbering Plan was created to standardize phone numbers and to assign area codes. It set the following rules for assigning area codes:

- Area codes are 3 digits long.
- The first digit cannot be a 0 or a 1.
- The second digit must be a 0 or a 1.
- Sixteen area codes are reserved for special services. (You might be familiar with some of these area codes, such as 411, 911, 800, and 900.)

How many area codes were available under these rules?

**4.** In 1994, the last area code was assigned under these rules. The third rule has now been changed to allow any number as the second digit of an area code. How many area codes are available with this change? Explain how you found this number.

**5.** Approximately how many phone numbers, including the area code, are available now under the North American Numbering Plan? Are enough phone numbers available? Explain why or why not.

**6.** What is the probability that a phone number will end in 7? In 77? In a 5 or a 6?

**7.** What is the probability that a phone number will end in an odd number? What is the probability that a randomly assigned phone number, excluding the area code, will have all odd digits?

**8.** What is the probability that you would be assigned one of these phone numbers in your area code: 123-MATH or 234-MATH?

**9.** If you moved to a town with a new area code, what is the probability that you would be assigned the same seven-digit phone number you have now?

## References

AT&T Archives and AT&T Bell Laboratories. *Highways, Hubs, and Networks.* New York: Scholastic, 1995.

Bell Telephone Laboratories. *Engineering and Operations in the Bell System*. Indianapolis, Ind.: Bell Telephone Laboratories, 1983.

Hochheiser, S., AT&T historian. □

*(Answers on page 176)*

## Mathematics Detective: Hints and Solutions

*(Continued from page 175)*

**1.** $8 \times 8 \times 10 \times 10 \times 10 \times 10 \times 10 = 6{,}400{,}000$.

**2.** Remember, 0 and 1 still cannot be used as the first digits. (The digit 0 at the beginning of a number signals the operator, and the digit 1 signals a long-distance call.) $8 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 8{,}000{,}000$ phone numbers. 8,000,000 – 6,400,000 = 1,600,000 more numbers. Eight million numbers is enough for most locations but not for very large cities because both families and businesses need phone numbers, and some have more than one. Some area codes include several cities.

**3.** $8 \times 2 \times 10 = 160$ area codes. 160 – 16 reserved codes = 144 usable area codes.

**4.** $8 \times 10 \times 10 = 800$ area codes. 800 – 16 reserved codes = 784 usable area codes.

**5.** The United States has 784 area codes and 8 million numbers in each area code. Multiplying these numbers gives 6,272,000,000. For comparison, this number is close to the population of the world! Currently, enough telephone numbers are available. Keep in mind that some families do not have phones, and some families have several phone numbers. Businesses may have one number or many.

**6.** Ten possibilities exist for the last digit of a phone number; therefore, the probability of a number ending in 7 is 1/10. The probability of a number ending in 77 is $P(\textit{end in 77}) = 1/10 \times 1/10 = 1/100$. Alternatively, 100 combinations are possible for the last two digits (00–99 inclusive), and 77 is one of these possibilities, giving a probability of 1/100. The chances are 2 out of 10 that a number will end in 5 or 6; therefore, the probability is $2/10 = 1/5$.

**7.** 1/2. Half of the digits 0–9 are odd. The probability that all seven digits are odd is $1/2 \times 1/2 \times 1/2 \times 1/2 \times 1/2 \times 1/2 \times 1/2 = 1/128$.

**8.** $P(\textit{123-MATH}) = 0$ because a phone number cannot begin with 1. $P(\textit{234-MATH}) = 1/8 \times 1/10 \times 1/10 \times 1/10 \times 1/10 \times 1/10 \times 1/10 = 1/8{,}000{,}000$. Alternatively, we know from problem 2 that 8,000,000 phone numbers are possible, and 234-MATH is one of these.

**9.** The probability of getting one particular phone number is 1/8,000,000. □