

## Taking Its Toll

NAME \_\_\_\_\_

To complete this activity, you will need a toll ticket from a road in your area or access to a web site that provides the same information. Use that information to answer the questions below.

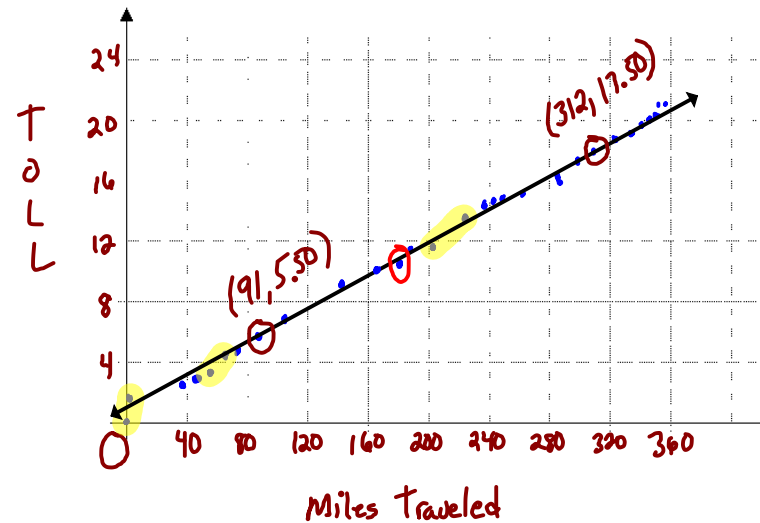
Justify your answers, and show any mathematical work.

### Gather the Data

1. Determine the cumulative number of miles from the beginning of the route to each successive toll plaza. Use the *Toll Data Recording Sheet* to record the total distance.
2. Determine the cumulative toll at each exit from the start of the route. Use the *Toll Data Recording Sheet* to record the total toll.

### Make a Graph

3. Make a graph of miles traveled ( $x$ ) vs. toll ( $y$ ). Be sure to include a title on the graph, and label the  $x$ -axis and  $y$ -axis. Each axis should have evenly spaced intervals that are numbered.



### Analyze the Graph

4. Imagine if lines were drawn between every pair of consecutive points. Which pair of points would have the steepest line connecting them? Determine the slope of the line between those two points. What does the steepest slope indicate about the toll between these two plazas?

$$\begin{matrix} (0, 0) & (2, 1.50) \\ x_1, y_1 & x_2, y_2 \end{matrix} \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.50 - 0}{2 - 0} = \frac{1.50}{2} = \frac{.75}{1}$$

Most expensive part of the road 75¢ for each mile

5. Are there two points on the graph such that the line connecting them has a slope of zero? If so, list the points below, and explain how you know that the slope is zero. If not, explain what must be true for a line to have a zero slope. What does a zero slope indicate about the toll between the two plazas?

$$(358, 21.25) (359, 21.25)$$

Free part of the road 1 mile

6. On the graph, draw a line that approximates the data. Your line should be drawn so that roughly half the points lie above the line and half the points lie below the line. This line is known as a *line of best fit*.

- a) What is the y-intercept of the line of best fit?

$$y = mx + b \quad 5.50 = \frac{1}{18.41}(91) + b$$

$$5.50 = 4.94 + b$$

$$-4.94 \quad -4.94$$

$$b = .56$$

- b) What is the slope of the line of best fit? Recall that  $m = \frac{y_2 - y_1}{x_2 - x_1}$ .

$$\begin{matrix} (91, 5.50) & (312, 17.50) \\ x_1, y_1 & x_2, y_2 \end{matrix} \quad \frac{17.50 - 5.50}{312 - 91} = \frac{12.00}{221} = \frac{12}{221}$$

- c) Write an equation for the line of best fit.

$$\frac{1}{18.41} \text{ slope}$$

$$y = \frac{1}{18.41}x + .56$$

7. Test your equation from Question 6c using one of the points from the graph. That is, substitute the x-coordinate of a point for x in your equation, and solve for y. Is the result close to the actual value of the y-coordinate? Explain any difference.

$$y = \frac{1}{18.41}x + .56 \quad (180, 10.50) \quad 10.50 = \frac{1}{18.41}(180) + .56$$

8. In the context of this problem, what does the slope of the line of best fit represent?

Cost is \$1 for every 18.41 miles

9. If the road continued for 1000 miles, how much would the toll be?

$$y = \frac{1}{18.41}(1000) + .56 \quad \$54.88$$

10. How would you determine the amount of the toll if a new plaza were added between two existing plazas? Describe your strategy. Then add a plaza between any two plazas and apply your strategy. Where did you add the plaza, and what is the amount of the toll at the new plaza?

Determine the number of miles from zero for the plaza then use the equation to determine the cost of the toll for that distance

Ex. For a toll plaza at 130 placed between

Exit numbers 110 and 146

$$y = \frac{1}{18.41}(130) + .56$$

$$y = 7.62$$

Cost = \$7.62 for a toll plaza at exit 130



## Exit Fares on the Pennsylvania Turnpike

EXIT NUMBER	TOLL
2	\$1.50
39	2.50
48	3.00
57	3.50
67	4.25
75	4.50
91	5.50
110	6.50
146	8.50
161	9.50
180	10.50
189	11.00
201	11.75
226	13.25
236	13.75
242	14.00
247	14.25
266	15.00
286	16.25
298	16.75
312	17.50
326	18.50
333	19.00
339	19.50
343	20.00
351	20.50
358	21.25
359	21.25

*Note that exit numbers on the PA Turnpike correspond to mileage.  
For example, Exit 247 occurs 247 miles from the Ohio-Pennsylvania border.*